Shrinking Candles, Running Water, Folding Boxes

This activity allows students to look for functions within a given set of data. After analyzing the data, the student should be able to determine a type of function that represents the data. This lesson plan is adapted from an article by Jill Stevens that originally appeared in the September 1993 issue of the Mathematics Teacher.

Learning Objectives

Students will be able to:

- learn to analyze data to determine the type of function that most closely matches the data
- demonstrate an understanding of how modifying parameters will change the graphs of functions by writing equations for those functions

Materials

- Activity Sheet 1
- Activity Sheet 2
- M&M's
- Paper
- Plates, cups, or other containers
- Stopwatches
- Birthday candles
- Matches
- Heat-resistant tiles or plates
- Rulers
- Centimeter grid paper
- Weather data
- Metersticks
- Jug or coffee pot with a spigot
- Scissors
- Graphing calculators

Instructional Plan

The *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) calls for the continued study of functions so that all students can-

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions. (NCTM 1989, 154)

The curriculum standards also stress the continued study of data analysis and statistics so that all students can-

- construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
- use curve fitting to predict from data. (NCTM 1989, 167)

The following activities promote these objectives.

Prerequisites: Students should understand the relationships between various functions and their graphs. Their knowledge should cover linear, quadratic, higher-degree-polynomial, exponential, and trigonometric functions. They should also be able to use a graphing calculator with features that will allow them to edit data, create scatterplots, and draw functions over those plots.

Directions: The activities will require three to five class periods to complete with students working in groups of three to four. The following problem will introduce the activities and allow time for any necessary instruction on using a graphing calculator. One explanatory example is usually sufficient for students successfully to use a graphing calculator for these activities.

Allow time for students to explain why they chose a particular function

As an example, our class discussed a unit called "How Does Corn Grow?" to familiarize students with data and graphing. Students planted some corn. After the seeds sprouted, they measured their growth each day. Analyze the collected data.

Day 1 is the first day that sprouts appeared. Each group chose a sprout and measured its height for five days. The following results from one group are shown.

Day	Height (cm)
1	1.50
2	2.59
3	3.91
4	6.02
5	7.11

Ask students to enter the data into a graphing calculator, make a scatterplot of the data on the calculator, and analyze the results.

Before making the scatterplot, students must decide on an appropriate domain and range so that all their points will appear on the graph. In this example, we have chosen the day number as the independent variable (*x*) and the height of the corn plant as the dependent variable (*y*). We will let the *x*-axis go from -5 to 10 and the *y*-axis go from -5 to 15. In so doing, all data will appear on the screen so that both axes can be seen. After setting the appropriate domain and range, students should use the calculator to produce a scatterplot of the data and analyze the results.

Students can discuss how their equations can predict future function values

Students should decide what type of function the data most closely represents and write an equation to fit the data. In this example, the data appear to be linear. The students can make an "eyeball" fit of the data by examining the scatterplot and determining a y-intercept and slope. They might try a *y*-intercept of 0.05 and a slope of 1.3. The students should use the calculator to draw the equation $y = 1.3x + 0.05$ over the scatterplot to check the accuracy of the fit, which is shown in **Figure 1**. The equation appears to fit the data closely. In many examples, students will try several equations before they find a close fit.

Figure 1

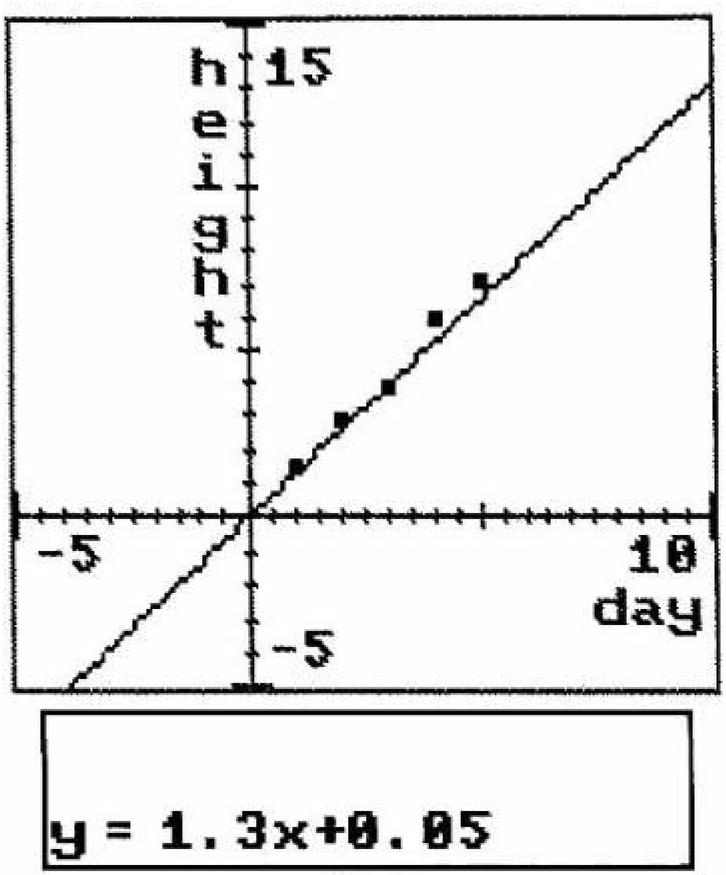

Fig 1.
Graph and equation from "How Does Corn Grow?" example; x scl = 0.5, y scl = 1.

Allow time for students to explain why they chose a particular type of function, how they derived their equation, and what restrictions the problem situation places on the domain and range of the function. In our example, *x* and *y* can be any real number in the algebraic equation, but in the model, *x* must be a positive integer and y must be a positive real number. Also, an upper bound will be necessary for *x* and *y*, since the corn will not continue to grow indefinitely.

The students should also discuss how their equation could be used to predict future function values. They could use their equation to predict the height of a corn sprout on future days. If they had actually grown the corn plants, they could test their predictions by later measuring the specific sprouts.

The same general procedure that was outlined in the example problem should be followed for the activity sheets. The student should collect data, produce a scatterplot, analyze it, choose a function, and write an equation. Test the equations by drawing the function over the scatterplot on the graphing calculator. Class discussion should include why a certain function type was chosen, whether another function type might also work, how equations were derived, what restrictions are placed on the domain and range by the problem situation, and whether the model could be used to predict future values. Although no *one* answer is correct for each problem, some answers may be better than others. Some sample solutions are given.

Answers Sheet 1: Figure 2 graphs the following data for "EliM&Mination" and sample solution 1.

Trial Number	**Number Remaining**
1	126
2	72
3	39
4	20
5	10
6	4
7	1
8	1
9	0

Figure 2

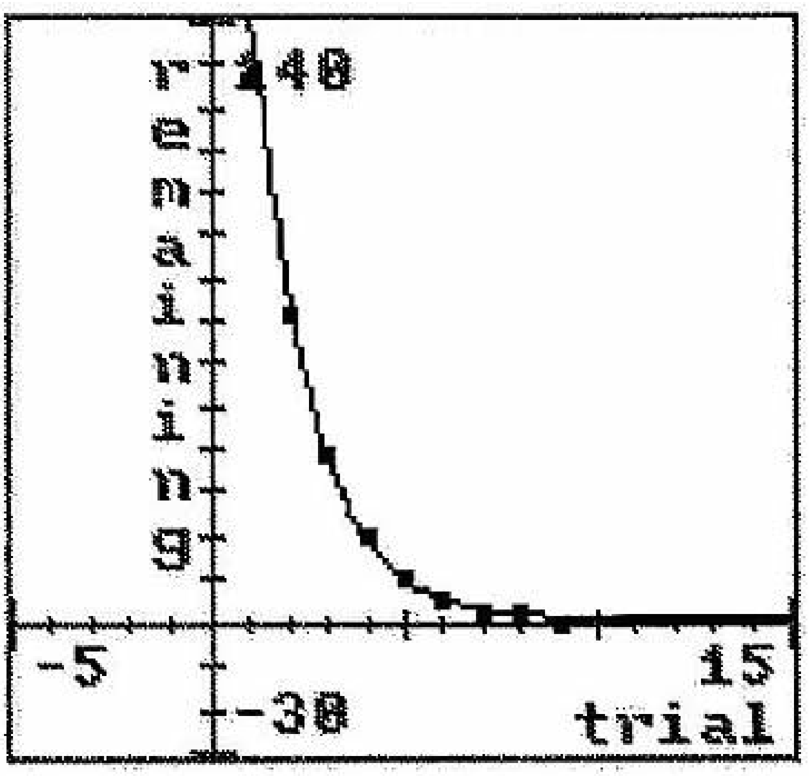

Fig. 2
M&M's activity graph; $y = 300\,(0.5)^x$.
x scl = 1, y scl = 10.

Figure 3 gives another view of data from sample solution 2.

Trial Number	Number Remaining
1	125
2	65
3	33
4	15
5	9
6	8
7	6
8	2

9	0

Figure 3

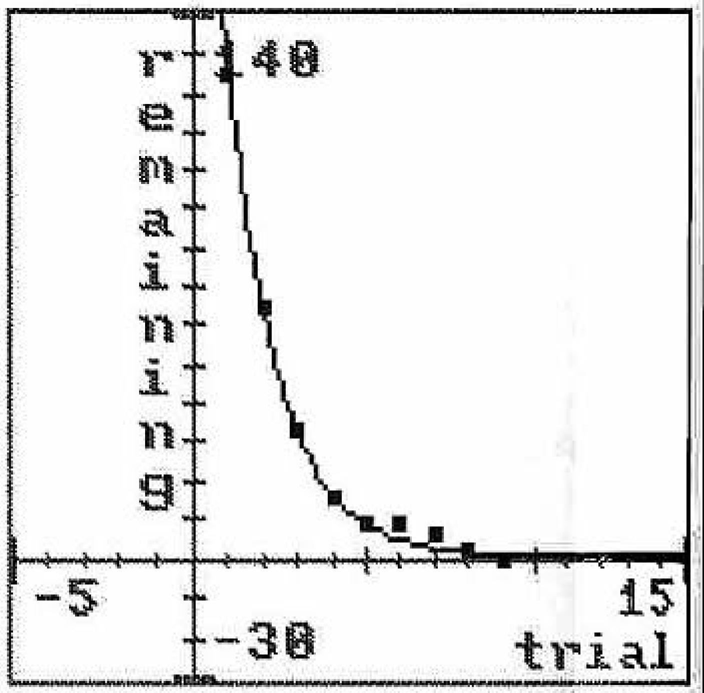

Fig. 3
Another solution to M&M's problem
$y = (0.5)^{(x-5)}$.
x scl = 1, y scl = 10.

A possible solution to "The Flaming Function" example graphs the data presented here (see **figure 4**).

Time (sec)	Height (cm)
0	4.6
20	4.3
40	4.1
60	3.9
80	3.6
100	3.4
120	3.3

140	3.2
160	3.1
180	3.0
200	2.8
220	2.7
240	2.5
460	2.3
280	2.1
300	1.9

Figure 4

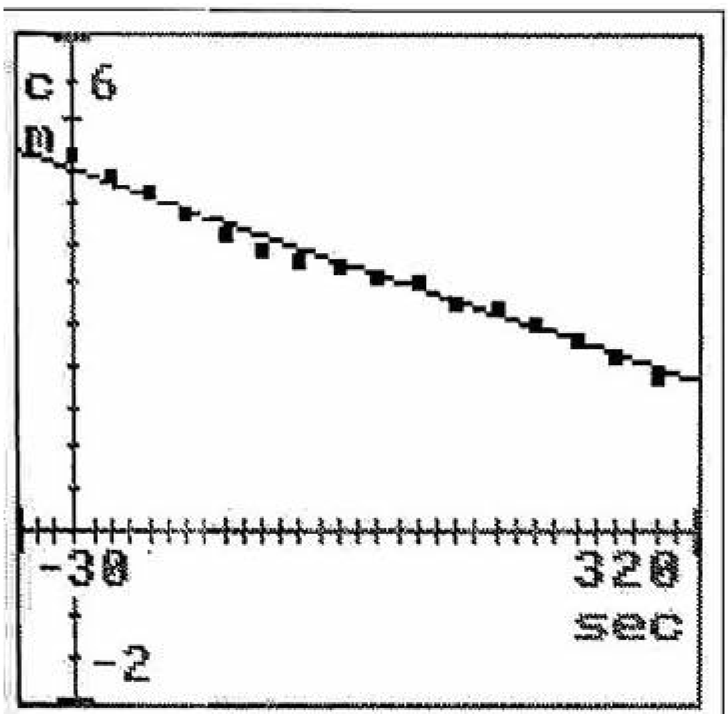

Fig. 4
"The Flaming Function's" possible solution;
$y = (-1/125)x + 4.4$
x scl = 10, y scl = 0.5

Answers Sheet 2: **Figure 5** graphs these data for "All Boxed In" sample solution.

Height (cm)	Volume (cm 3)
1	324
2	512
3	588
3.5	591.4
4	576
5	500
6	384
7	252
8	128
9	36

Figure 5

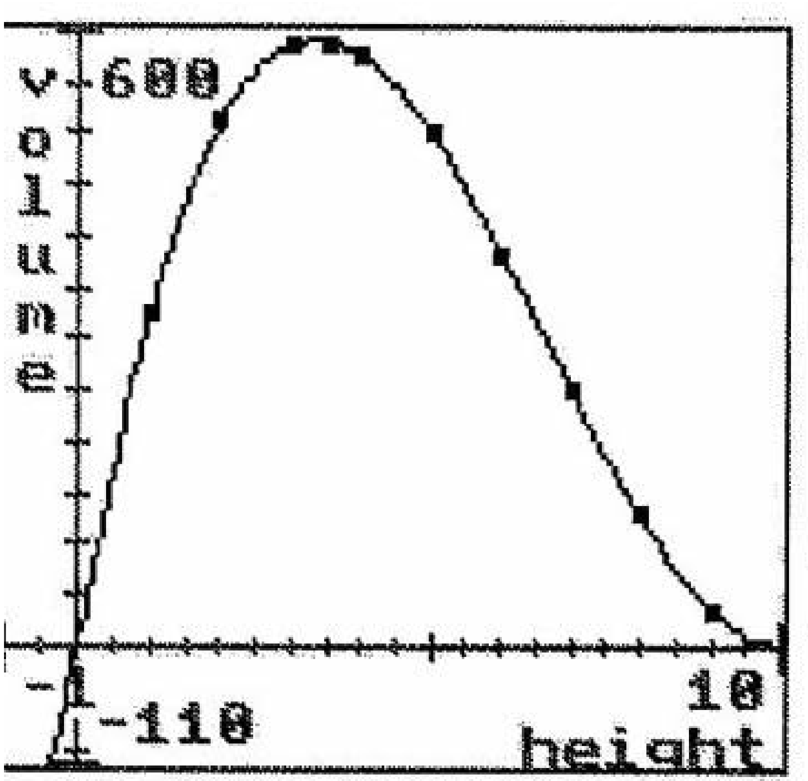

Fig. 5
"All Boxed In" problem is graphed
with $y = x(20 - 2x)^2$.
x scl = 0.5, y scl = 50.

" 'Weather' It's a Function" was calculated from the normal high temperature for the Dallas area and was obtained by writing to a local television weather forecaster; see **Figure 6**. The average monthly temperature for many cities can be found in almanacs. This information reduces the domain to twelve values but still yields a fairly smooth curve. Be sure the calculator is in the radian mode to work this problem.

Day	Temperature
1	55
15	53
32	56
46	59
60	63
74	67

91	72
105	77
121	81
135	84
152	89
196	98
213	99
227	98
244	94
258	90
274	85
288	80
305	72
319	66
335	61
349	58

Figure 6

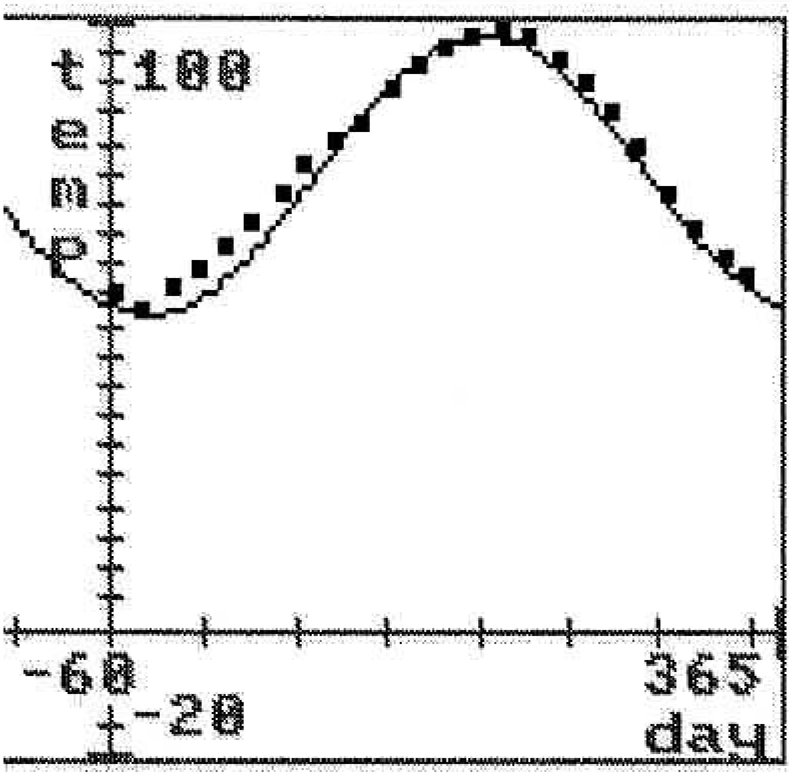

Fig. 6
" 'Weather' It's a Function" graph of a trigonometric sine wave: $y = 23 \sin(0.017214x - 115) + 75$. x scl = 50, y scl = 5.

"Water Level" calculations are presented here. See the resulting graph in **Figure 7**.

Time (sec)	Depth (mm)
0	249
15	219
30	182
45	144
60	110
75	80
90	58
105	36

120	24
135	17

Figure 7

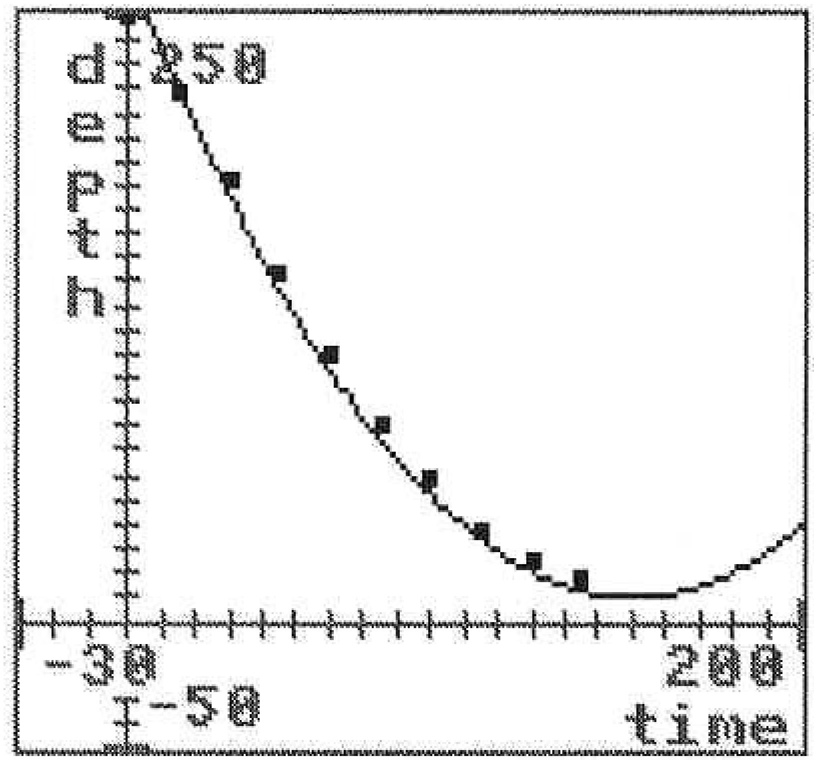

Fig. 7
The "Water Levels" problem's graphed quadratic equation shows that $y = 0.0115\,(x - 150)^2 + 10$.
x scl = 10, y scl = 10.

Discussion and extension activities: The "EliM&Mination" problem serves as a model of decay. Have students suggest real-life situations that this problem might model.

The "Flaming Function" is linear. It is interesting to compare the equation the students obtain from the "eyeball" method with those from the median-median-line and linear-least-square methods of curve fitting. The Data Analysis software package from the National Council of Teachers of Mathematics (1988) is a good source for information about these methods.

"All Boxed In" presents a cubic model. This problem could be extended by asking students to find the length plus girth and the surface area of each box in addition to its volume. The problem would then include a linear, quadratic, and cubic model. The post office uses the concept of girth to limit the size of boxes sent in the mail (length + girth <= 108 inches). Ask students to find the box with the greatest volume for the least surface area that would meet postal regulations.

The problem " 'Weather' It's a Function" produces a sine wave. The students can use this model to predict temperatures and then check to see how close their predictions come to the actual members.

The "Water Level" problem yields data for a quadratic model. An interesting activity would be to have different groups use different-sized jugs and compare the curves obtained from the data.

If computers are available, students may want to print their graphs rather than draw them. The figures illustrated in this article were created using The Mathematics Exploration Toolkit (IBM 1988). Another source of software is Data Analysis (NCTM 1988), which allows students to print tables and graphs. The software also includes many algorithms to fit curves to data sets.

NCTM Standards and Expectations

Data Analysis & Probability 9-12

1. Compute basic statistics and understand the distinction between a statistic and a parameter.
2. Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

References

- International Business Machines Corporation. The Mathematics Exploration Toolkit. Boca Raton, Fla.: The Corporation, 1988. Software.

- National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: The Council, 1989.

- North Carolina School of Science and Mathematics (NCSSM). Data Analysis. Reston, Va.:National Council of Teachers of Mathematics, 1988.Software.

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